COMMUTATORS OF A_∞ STRUCTURES

TOM LADA

Dedicated to Prof. J. D. Stasheff on the occasion of his sixtieth birthday

1. Introduction

This note will be for the most part of an expository nature; many of the results here have appeared elsewhere. We would like however, to highlight the connection between Jim Stasheff's early work on A_∞ algebras in the 1960's and the notion of sh Lie algebras (L_∞ algebras) which is currently popular in mathematical physics; see, for example, [1] and [7]. This connection involves some work of Victor Gugenheim and Larry Lambe who were two of Jim's colleagues over the years. We will first review the concepts of cofree coalgebras and cofree cocommutative coalgebras; the next idea that we will require is that of a coderivation of such coalgebras. We will then relate these concepts to A_∞ and L_∞ algebras and show that "commutators on A_∞ algebras yield L_∞ algebras".

I would like to thank Jim Stasheff for igniting my interest in higher homotopy structures years ago and for encouraging a continuation of it. It has been an honor as well as a pleasure working with him and with some of his colleagues who are included in the bibliography of this note.

2. Coderivations

We will work with graded vector spaces over a field k of characteristic zero. As is the usual case in dealing with graded objects, we will systematically use the Koszul sign convention which results in the introduction of a factor of $(-1)^{pq}$ into a calculation whenever the order of two "things" is interchanged. For an ordered collection of graded objects $x_1, \ldots, x_n$ we denote the total Koszul sign by $e(\sigma)$ where $x_1, \ldots, x_n = e(\sigma)x_{\sigma(1)}, \ldots, x_{\sigma(n)}$ where $\sigma \in S_n$.

Recall that a coalgebra over k is a vector space A over k together with linear maps of degree zero $\Delta_A : A \longrightarrow A \bigotimes A$ and $\epsilon : A \longrightarrow k$ that satisfy the equalities $(\Delta \otimes id_A) \circ \Delta = (id_A \otimes \Delta) \circ \Delta$ and $(id_A \otimes \epsilon) \circ \Delta = id_A = (\epsilon \otimes id_A) \circ \Delta$ The reduced comultiplication $\bar{\Delta}$ is defined by the equation $\Delta x = 1 \otimes x + x \otimes 1 + \bar{\Delta} x$. We will use the reduced comultiplication in the rest of this note and denote it by the same symbol as the comultiplication.

We next recall that a coderivation of a coalgebra A is a linear map $f : A \longrightarrow A$ such that $\pi_0 f = 0$ and

$$(f \otimes 1 + 1 \otimes f)\Delta_A = \Delta_A f.$$

Here π_n is the projection $\pi_n : A \longrightarrow A_n$.

The coalgebras that will be of interest to us are the cofree coalgebra on the graded vector space V, denoted by T^*V, with comultiplication denoted by Ψ and the cofree commutative coalgebra on V, denoted by $\bigwedge^* V$ with comultiplication denoted by

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Δ. We write $T^*V = \bigoplus T^nV$, the tensor coalgebra on V, and $\bigwedge^* V = \bigoplus \bigwedge^n V$ together with projection maps $\pi_n : T^*V \longrightarrow T^nV$ and $p_n : \bigwedge^* V \longrightarrow \bigwedge^n V$.

The reduced comultiplications are given explicitly by

$$\Psi(v_1 \otimes \cdots \otimes v_n) = \sum_i (v_1 \otimes \cdots \otimes v_i) \otimes (v_{i+1} \otimes \cdots \otimes v_n)$$

and

$$\Delta(v_1 \wedge \cdots \wedge v_n) = \sum_{1 \le j \le n-1} \sum_\sigma e(\sigma)(v_{\sigma(1)} \wedge \cdots \wedge v_{\sigma(j)}) \otimes (v_{\sigma(j+1)} \wedge \cdots \wedge v_{\sigma(n)})$$

where σ runs through all $(j, n-j)$ unshuffles.

These two coalgebras are related by an injective coalgebra map $\chi : \bigwedge^* V \longrightarrow T^*V$ given by

$$\chi(v_1 \wedge \cdots \wedge v_n) = \sum_{\sigma \in S_n} e(\sigma) v_{\sigma(1)} \otimes \cdots \otimes v_{\sigma(n)};$$

denote the inverse of χ (on the image of χ) by ρ. See [5] for a more detailed discussion of these concepts.

One may in fact regard $\bigwedge^* V$ as the subcoalgebra of T^*V that consists of the vector subspace of symmetric tensors in T^*V. The comultiplication Δ may then be regarded as the restriction of Ψ to this subspace. A very complete description of this point of view, as well as a thorough discussion of other subcoalgebras and coderivations thereof is contained in Lars Kjeseth's dissertation [3]. We prefer to take a more universal approach in this note but we will point out the subcoalgebra approach when profitable.

In [2] Gugenheim and Lambe sketch the construction of coderivations on the tensor coalgebra. Let us review those ideas next and then show that they carry over to cocommutative coalgebras by using the symmetrization map. The comultiplication $\Psi : T^*V \longrightarrow T^*V \bigotimes T^*V$ may be defined by the commutativity of the diagram

$$\begin{array}{ccc} T^*V & \xrightarrow{\Psi} & T^*V \bigotimes T^*V \\ \pi_{a+b} \downarrow & & \downarrow \pi_a \otimes \pi_b \\ T^{a+b}V & \xrightarrow{F(a,b)} & T^aV \bigotimes T^bV \end{array}$$

where $F(a,b) : T^{a+b}V \longrightarrow T^aV \bigotimes T^bV$ is the canonical isomorphism; denote its inverse by $E(a,b)$ when needed.

A coderivation $f : T^*V \longrightarrow T^*V$ may be characterized by

$$\pi_n f = \sum E(1,\ldots,1)(\pi_1 \otimes \cdots \otimes \pi_1 f \otimes \cdots \otimes \pi_1)\Psi^{(n)}$$

where

$$\Psi^{(n)} = (\Psi \otimes \underbrace{1 \otimes \cdots \otimes 1}_{n-2})) \circ \cdots \circ (\Psi \otimes 1) \circ \Psi$$

and

$$E(1,\ldots,1) : T^1V \otimes \cdots \otimes T^1V \longrightarrow T^nV$$

is the isomorphism given by

$$E(1,\ldots,1) = E(n-1,1) \circ E(n-2,1) \otimes 1 \circ \cdots \circ E(1,1) \otimes \underbrace{1 \otimes \cdots \otimes 1}_{n-2}.$$

If $f : T^nV \longrightarrow V$ is a linear map, it may be extended to a coderivation $\hat{f} : T^*V \longrightarrow T^*V$ by setting $\hat{f}(v_1 \otimes \cdots \otimes v_k) = 0$ for $k < n$ and

$$\hat{f}(v_1 \otimes \cdots \otimes v_k) = \sum_{i=1}^{k-n+1} (1^{\otimes i-1} \otimes f \otimes 1^{\otimes k-n+1-i})(v_1 \otimes \cdots \otimes v_k)$$

for $k \geq n$. This follows from the general description of coderivations above.

We next show that the above description of the comultiplication and coderivations on T^*V passes over to the cocommutative setting.

Lemma 1. *$\Delta : \bigwedge^* V \longrightarrow \bigwedge^* V \bigotimes \bigwedge^* V$ is characterized by the commutative diagram*

$$\begin{array}{ccccc}
\bigwedge^* V & \xrightarrow{\Delta} & \bigwedge^* V \bigotimes \bigwedge^* V & \xrightarrow{\chi\otimes\chi} & T^*V \bigotimes T^*V \\
p_{a+b} \downarrow & & & & \downarrow \pi \otimes \pi \\
\bigwedge^{a+b} V & \xrightarrow{\chi} & T^{a+b}V & \xrightarrow{F(a,b)} & T^aV \bigotimes T^bV
\end{array}$$

Proof. This follows from the diagram

$$\begin{array}{ccccc}
\bigwedge^* V & \xrightarrow{\Delta} & \bigwedge^* V \bigotimes \bigwedge^* V & \xrightarrow{\chi\otimes\chi} & T^*V \bigotimes T^*V \\
=\downarrow & & & & \downarrow= \\
\bigwedge^* V & \xrightarrow{\chi} & T^*V & \xrightarrow{\Psi} & T^*V \bigotimes T^*V \\
p_{a+b} \downarrow & & \pi_{a+b} \downarrow & & \pi_a \otimes \pi_b \downarrow \\
\bigwedge^{a+b} V & \xrightarrow{\chi} & T^{a+b}V & \xrightarrow{F(a,b)} & T^aV \bigotimes T^bV
\end{array}$$

where the top rectangle commutes because χ is a coalgebra map, the lower left square commutes because χ is injective and the lower right square commutes by the definition of Ψ. □

Lemma 2. *Let $\Delta^{(n)} : \bigwedge^* V \longrightarrow T^n \bigwedge^* V$ be the iterated comultiplication and let $\chi : \bigwedge^n \bigwedge^* V \longrightarrow T^n \bigwedge^* V$ be as above. Then Image$(\Delta^{(n)}) \subset$ Image(χ).*

Proof. Suppose that $I_1 \cup \cdots \cup I_n$ is a partition of the index set $I = \{1, 2, \ldots, N\}$ so that $v_{I_1} \otimes \cdots \otimes v_{I_n} \in Im(\Delta^{(n)})$; here, $v_{I_k} = v_{k_1} \wedge \cdots \wedge v_{k_j}$ for $k_1, \ldots, k_j \in I_k$. We claim that $v_{I_{\sigma(1)}} \otimes \cdots \otimes v_{I_{\sigma(n)}} \in Im(\Delta^{(n)}$ for all $\sigma \in S_n$. To see this, consider $v_{I_1 \cup \cdots \cup I_n} \in \bigwedge^* V$. Apply Δ to obtain

$$\Delta(v_{I_1 \cup \cdots \cup I_n}) = \sum e(\sigma) v_{J_1} \otimes v_{J_2}$$

where the sum is taken over all unshuffles (J_1, J_2) of I. Among the summands are the n terms of the form

$$v_{I - I_{k_1}} \otimes v_{I_{k_1}}$$

for $k_1 = 1, \ldots, n$. Apply $\Delta \otimes 1$ to each of these summands and obtain terms which include $n-1$ terms of the form

$$v_{I - I_{k_1} - I_{k_2}} \otimes v_{I_{k_2}} \otimes v_{I_{k_1}}$$

with $k_1 \neq k_2$. Now apply $\Delta \otimes 1 \otimes 1$ to each of these $n(n-1)$ terms and select from the result the $n-2$ terms of the form

$$v_{I - I_{k_1} - I_{k_2} - I_{k_3}} \otimes v_{I_{k_3}} \otimes v_{I_{k_2}} \otimes v_{I_{k_1}}.$$

Continue this procedure, which terminates with the application of $\Delta \otimes 1 \otimes \cdots \otimes 1$, to obtain the $n!$ terms of the form

$$v_{I-I_{k_1}-\cdots-I_{k_{n-1}}} \otimes v_{I_{k_{n-1}}} \otimes \cdots \otimes v_{I_{k_1}}.$$

It is clear that these are all of the permutations of the original term. □

Lemma 3. *The projection $p_n : \bigwedge^* V \longrightarrow \bigwedge^n V$ is given by the composition*

$$p_n = \rho E(1, \dots, 1)\chi^{\otimes n}(p_1 \otimes \cdots \otimes p_1)\Delta^{(n)}.$$

Proof. By [2] we have

$$\pi_n = E(1, \dots, 1)(\pi_1 \otimes \cdots \otimes \pi_1)\Psi^{(n)}$$

and, because $p_n = \rho\pi_n\chi$ and $\pi_1\chi = \chi p_1$ by Lemma 1,

$$\begin{aligned} p_n &= \rho E(1, \dots, 1)(\pi_1 \otimes \cdots \otimes \pi_1)\Psi^{(n)}\chi \\ &= \rho E(1, \dots, 1)(\pi_1 \otimes \cdots \otimes \pi_1)\chi^{\otimes n}\Delta^{(n)} \\ &= \rho E(1, \dots, 1)\chi^{\otimes n}(p_1 \otimes \cdots \otimes p_1)\Delta^{(n)} \end{aligned}$$

□

Now, if $f : \bigwedge^* V \longrightarrow \bigwedge^* V$ is a coderivation, the equation

$$(f \otimes 1 + 1 \otimes f)\Delta = \Delta f$$

is equivalent to

$$(\pi_a \otimes \pi_b)(\chi \otimes \chi)(f \otimes 1 + 1 \otimes f) = (\pi_a \otimes \pi_b)(\chi \otimes \chi)\Delta f = (\pi_a \otimes \pi_b)\Psi\chi f$$

because χ is an algebra map; this in turn is equal to $F(a, b)\pi_{a+b}\chi f$ by [2] which then equals $F(a, b)\chi p_{a+b} f$.

As a result,

$$\chi p_{a+b} f = E(a, b)(\pi_a \otimes \pi_b)(\chi \otimes \chi)(f \otimes 1 + 1 \otimes f)\Delta$$

or, because the right hand side is in the image of Δ,

$$p_{a+b} f = \rho E(a, b)(\pi_a \otimes \pi_b)(\chi \otimes \chi)(f \otimes 1 + 1 \otimes f)\Delta.$$

Finally, we have

Lemma 4. *A coderivation $f : \bigwedge^* V \longrightarrow \bigwedge^* V$ is characterized by*

$$p_n f = \rho E(1, \dots, 1)\chi^{\otimes n}(\sum p_1 \otimes \dots p_1 f \otimes \cdots \otimes p_1)\Delta^{(n)}.$$

Proof. We can see that $p_2 f$ satisfies the claim by letting $a = b = 1$ in the previous remark. By induction on n, we may write

$$\begin{aligned} p_{n+1} f &= \rho E(n, 1)(\pi_n \otimes \pi_1)(\chi \otimes \chi)(f \otimes 1 + 1 \otimes f)\Delta \\ &= \rho E(n, 1)(\chi \otimes \chi)(p_n \otimes p_1)(f \otimes 1 + 1 \otimes f)\Delta \\ &= \rho E(n, 1)(\chi \otimes \chi)(p_n f \otimes p_1)(f \otimes 1 + p_n \otimes p_1 f)\Delta \\ &= \rho E(n, 1)(\chi \otimes \chi)[\rho E(1, \dots, 1)\chi^{\otimes n}(\sum p_1 \otimes \dots p_1 f \otimes \cdots \otimes p_1)\Delta^{(n)} \otimes p_1 \\ &\quad + \rho E(1, \dots, 1)\chi^{\otimes n}(p_1 \otimes \cdots \otimes p_1)\Delta^{(n)} \otimes p_1 f]\Delta \\ &= \rho E(n, 1)(\chi \otimes \chi)(\rho \otimes 1)(E(1, \dots, 1) \otimes 1)\chi^{\otimes n} \otimes 1)[(\sum p_1 \otimes \dots p_1 f \otimes \cdots \otimes p_1) \otimes p_1 \\ &\quad + (p_1 \otimes \cdots \otimes p_1) \otimes p_1 f](\Delta^{(n)} \otimes 1)\Delta \\ &= \rho E(n + 1, 1)\chi^{\otimes n+1}[\sum p_1 \otimes \cdots \otimes p_1 f \otimes \cdots \otimes p_1]\Delta^{(n+1)} \end{aligned}$$

after some minor simplifications. □

Remark: If $g : \bigwedge^* V \longrightarrow V$ is a linear map, then the extension of g to a coderivation $\hat{g} : \bigwedge^* V \longrightarrow \bigwedge^* V$ with $p_1\hat{g} = g$ is given by the equation in the preceding lemma. The perhaps more familiar formula for the extension of g to a coderivation where $g : \bigwedge^n V \longrightarrow V$ is non-zero for only n-fold wedges,

$$\hat{g}(v_1 \wedge \dots v_k) = \sum_\sigma e(\sigma) g(v_{\sigma(1)} \wedge \dots \wedge v_{\sigma(n)}) \wedge v_{\sigma(n+1)} \wedge \dots \wedge v_{\sigma(k)}$$

as σ runs through all $(n, k-n)$ unshuffles, can also be easily derived ¿from the preceding lemma.

Example: A familiar example of the construction in the lemma may be seen in the following situation. Suppose that $g : V \bigwedge V \longrightarrow V$ is a linear map (e.g. a Lie bracket). Regard g as a map $g : \bigwedge^* V \longrightarrow V$ by defining $g| \bigwedge^n V = 0$ when $n \neq 2$. To define $\hat{g} : \bigwedge^3 V \longrightarrow \bigwedge^2 V$, we calculate

$$p_2\hat{g} = \rho E(1,1)(\chi \otimes \chi)(g \otimes p_1 + p_1 \otimes g)\Delta.$$

(Signs and degrees are suppressed here for clarity).

$$x \wedge y \wedge z \xrightarrow{\Delta} (x\wedge y)\otimes z + (x \wedge z)\otimes y + (y\wedge z)\otimes x + z\otimes(x\wedge y) + y \otimes (x\wedge z) + x\otimes(y\wedge z)$$

$$\xrightarrow{g\otimes p_1 + p_1 \otimes g} g(x\wedge y)\otimes z + g(x\wedge z)\otimes y + g(y\wedge z)\otimes x + z\otimes g(x\wedge y) + y\otimes g(x\wedge z) + x \otimes g(y\wedge z)$$

$$\xrightarrow{\rho(\chi\otimes\chi)} g(x\wedge y)\wedge z + g(x\wedge z)\wedge y + g(y\wedge z)\wedge x$$

which is the form of the familiar Jacobi expression for Lie algebras. Note that here χ is the identity map on $\bigwedge^1 V = V$.

Coderivations on cofree cocommutative coalgebras are related to coderivations on cofree coalgebras by the following proposition.

Proposition 5. *Suppose that $f : T^*V \longrightarrow V$ is a linear map which extends to the coderivation $\hat{f} : T^*V \longrightarrow T^*V$. Then the diagram*

$$\begin{array}{ccccc} \bigwedge^* V & \xrightarrow{\chi} & T^*V & \xrightarrow{=} & T^*V \\ \widehat{f\circ\chi}\uparrow & & \hat{f}\uparrow & & \downarrow \pi_1 \\ \bigwedge^* V & \xrightarrow{\chi} & T^*V & \xrightarrow{f} & V \end{array}$$

commutes. Here, $\widehat{f\circ\chi}$ is the extension of the map $f\circ\chi : \bigwedge^* V \longrightarrow V$ to the coderivation $\widehat{f\circ\chi} : \bigwedge^* V \longrightarrow \bigwedge^* V$.

Proof. To compute $\chi\widehat{f\circ\chi}$, we compute

$$\pi_n\chi\widehat{f\circ\chi} = \chi p_n \widehat{f\circ\chi}$$

$$= \chi\rho E(1,\dots,1)\chi^{\otimes n}[\sum p_1 \otimes \dots \otimes p_1\widehat{f\circ\chi}\otimes p_1 \otimes \dots \otimes p_1]\Delta^{(n)}$$

$$= E(1,\dots,1)[\sum \chi p_1 \otimes \dots \otimes \chi f \chi \otimes \dots \otimes \chi p_1]\Delta^{(n)}$$

$$= E(1,\dots,1)[\sum \pi_1\chi \otimes \dots \otimes \chi f\chi \otimes \dots \otimes \pi_1\chi]\Delta^{(n)}$$

$$= E(1,\dots,1)[\sum \pi_1 \otimes \dots \otimes \pi_1\hat{f} \otimes \dots \otimes \pi_1]\chi^{\otimes n}\Delta^{(n)}$$

(because $\chi f = f$ and $\pi_1\hat{f} = f$)

$$= E(1,\dots,1)[\sum \pi_1 \otimes \dots \otimes \pi_1\hat{f} \otimes \dots \otimes \pi_1]\Psi^{(n)}\chi$$

$= \pi_n\hat{f}\chi$. □

3. A_∞ and L_∞ Algebras

We can use the framework of the preceding section to show that the usual relationship between associative algebras and Lie algebras holds in a homotopy context.

First recall the definition of A_∞ Algebra given by Stasheff in [6].

Definition 6. *An A_∞ structure on a graded vector space V is a collection of linear maps $m_k : \bigotimes^k V \longrightarrow V$ with the degree of $m_k = k-2$. These maps are required to satisfy the identity*

$$\sum_{\lambda=0}^{n-1}\sum_{k=1}^{n-\lambda} \alpha m_{n-k+1}(a_1,\ldots,a_\lambda, m_k(a_{\lambda+1},\ldots,a_{\lambda+k}), a_{\lambda+k+1},\ldots,a_n) = 0$$

where $\alpha = (-1)^{k+\lambda+k\lambda+kn+k(|a_1|+\cdots+|a_\lambda|)}$.

Note that m_1 is a differential for V, m_2 is a multiplication, and the m_k's are higher associating homotopies.

In a similar fashion, we have [4], [5]

Definition 7. *An L_∞ structure on a graded vector space V is a collection of linear, skew symmetric maps $l_k : \bigotimes^k V \longrightarrow V$ of degree $k-2$ that satisfy the relation*

$$\sum_{i+j=n+1}\;\sum_{unsh(i,n-i)} e(\sigma)(-1)^\sigma(-1)^{i(j-1)} l_j(l_i(v_{\sigma(1)},\ldots,v_{\sigma(i)}), v_{\sigma(i+1)},\ldots,v_{\sigma(n)}) = 0$$

where $1 \leq i, j$.

Again, we have that l_1 is a differential for V, l_2 is a bracket, and the l_k's are higher homotopies that relate to the Jacobi expression.

A classic result in Lie algebra theory is that the commutator on an associative algebra V gives rise to a Lie algebra structure on V; in the notation of the previous two definitions, this result may be written as $m_2 \circ \chi = l_2$. Of course the associativity of m_2 is required in order to yield the Jacobi identity for the bracket l_2. If m_2 is only associative up to homotopy, one expects that the L_∞ structure maps may be obtained from the A_∞ structure maps via $l_n = m_n \circ \chi$. Indeed, a proof of this fact appears in [5]. We will, however, present here a slightly different proof that will utilize the concepts developed in the preceding section.

The main difficulty in proving this result involves keeping track of the permutations and the signs that are involved in the statement, and in following them through the two defining relations. This can be greatly simplified by using a different point of view. In [6], Stasheff showed that an A_∞ structure on V is equivalent to the existence of a degree -1 coderivation $D : T^*sV \longrightarrow T^*sV$ with $D^2 = 0$. Here, sV is the suspension of the graded vector space V; it is defined by $(sV)_n = V_{n-1}$. Similarly, in [4] and in [5] it is shown that an L_∞ structure on V is equivalent to a degree -1 coderivation D' on $\bigwedge^* sV$ with $D'^2 = 0$. We finally put all of the pieces together to obtain

Theorem 8. *Suppose that $\{m_n\} : T^*V \longrightarrow V$ is an A_∞ structure on V. Then $\{m_n \circ \chi\}$ is an L_∞ structure on V.*

Proof. Recall that the A_∞ structure induces linear maps $T^n sV \longrightarrow sV$ which we will also denote by m_n. These maps extend to coderivations with the property that $D = \sum \widehat{m_n} : T^*sV \longrightarrow T^*sV$ is a differential. We show that $D' : \bigwedge^* sV \longrightarrow \bigwedge^* sV$

defined by $D' = \sum \widehat{m_n \circ} \chi$ is also a differential. The diagram,

$$\begin{array}{ccc} \bigwedge^* sV & \xrightarrow{\chi} & T^*sV \\ D' \uparrow & & \uparrow D \\ \bigwedge^* sV & \xrightarrow{\chi} & T^*sV \\ D' \uparrow & & \uparrow D \\ \bigwedge^* sV & \xrightarrow{\chi} & T^*sV \end{array}$$

which commutes by Proposition 5, yields $D'^2 = 0$ because $D^2 = 0$ and χ is injective.
□

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Mathematics Dept., Box 8205, North Carolina State University, Raleigh, N.C. 27695
E-mail address: lada@math.ncsu.edu